



Application of Coupled Stochastic Systems on the Effect of Seasonal Recapitalization

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Abstract: This study models a distressed bank under Central Bank of Nigeria (CBN) regulatory pressure where baseline valuation decays exponentially with multiplicative seasonality effect. Three analytical paths and theoretical expected means were simulated showing up and down horizontal trend lines instead of exponential growth. Sensitivity analysis for volatility and drift shows both plots look alike with up and down horizontal movements, unlike growth banks where volatility dominates. Results show distressed pattern trapped sideways oscillation with slow decay, indicating that both growth rate and volatility cannot rescue the bank under regulatory overhang. Implication is range-bound trading rather than buy-and-hold. The results provide insight into how regulatory policy propagates across banks, how diversification benefits change across regimes, and how risk managers can stress-test portfolios under policy uncertainty.

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1.1. Introduction

Recapitalization is typically modeled as a one-time regulatory shock. However, the 2024 CBN exercise showed that banks raise capital in seasons: Quarter-1 for rights issues, Quarter-4 for retained earnings boosting. This seasonal recapitalization creates periodic liquidity shocks that affect two coupled markets: equity (where shares are issued) and money market (where liquidity is withdrawn). However, stochastic systems with regime-switching are well suited to model this behavior because they allow parameters and initial conditions to change at a known or estimated switching time. The regime-switching framework captures both the immediate jump effect at and the subsequent evolution under correlated Brownian drivers. The results provide insight into how regulatory policy propagates across banks, how diversification benefits change across regimes,

and how risk managers can stress-test portfolios under policy uncertainty.

However, stock market prices have been examined in a wide manner by many scholars. For instance, [1] studied the stochastic analysis of stock market price model, and result showed that the proposed model is efficient for the production of stock prices. In a similar manner [2] studied the stochastic analysis of stock market expected returns. Their results of comparing the variances of four different stocks indicated that stock1 is the best among the stocks of diverse companies, which is consistent with the work of [3]. Also, [4] considered a stochastic analysis of stock market expected returns and growth-rates. In trying to stochastic model [5] considered the stability analysis of stochastic model for stock market prices and did analysis of the unstable nature of stock market forces applying a new differential equation model that can impact the expected

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returns of investors. While [6] suggested in their study, some analytical solutions of stochastic differential equations with respect to Martingale processes and discovered that the solutions of some SDEs are related to other stochastic equations with diffusion part. On the other hand, [7] looked at the numerical techniques of solving stochastic differential equations. Their results showed that if the discretization value N is increasing, the Euler-Maruyama and Milstein techniques were closed to exact solution. Also, [8] worked on the stability of both analytical and numerical solutions for non-linear stochastic delay differential equations with jumps and they observed that the compensated stochastic methods inherit stability property of the correct solution. [9] studied the solution of differential equations and stochastic differential equations of time varying investment returns and obtained precise conditions governing asset price returns. The proposed model showed an efficient and reliable multiplicative inverse trend series. As if that is not enough, [10] studied stochastic model of the fluctuations of stock market price and obtained precise conditions for determining the equilibrium price. On the other hand [11] examined the stability behaviors of stochastic differential equations (SDEs) driven by time changed Brownian motions. In another dimension [12] studied stochastic methods in practical delineation of financial models and suggested useful delineation of asset stock price and volatility.

The problem is that current models treat recapitalization as discrete and uncoupled. [18] describes it as a shock but does not model its seasonal periodicity. As a result, banks cannot anticipate the coupled effect: when Bank A raises capital, it drains liquidity from Bank B, raising interbank rates. This coupling is stochastic because investor appetite is random. The aim is to apply two coupled stochastic differential equations to model the effect of seasonal recapitalization on asset prices, liquidity and bank capital adequacy. The model will have a seasonal recapitalization function driving both systems. The gap is significant: no study in Nigeria has modeled recapitalization as a seasonal coupled stochastic process. [19] and [20] discuss regulatory interest rate shocks but not recapitalization seasonality. This research introduces seasonality into regulatory impact analysis.

This paper is prescribed as follows: Section 2.1 presents the mathematical preliminaries, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

2.1: Mathematical Preliminaries

At this juncture we present few basic definitions as touching this dynamic area of study, hence we have as follows:

Definition 1: Probability space: This is a triple $(\Omega, \mathcal{F}, \tilde{A})$

where Ω represents a set of sample space, $\mathcal{F}$ represents a collection of subsets of Ω , while $\tilde{A}$ is the probability

measure defined on each event $A \in \mathcal{F}$. The collection $\mathcal{F}$ is a σ -algebra or σ -field such as $\Omega \in \mathcal{F}$ and $\mathcal{F}$ is closed under the arbitrary unions and finite intersections. Hence it is called probability measure when the following condition holds.

$$(i) P(A) \geq 0 \text{ for all } A \subset \Omega$$

$$(ii) P(\Omega) = 1$$

$$(iii) A, B \subset \Omega, A \cap B = \phi \text{ then } P(A \cup B) = P(A) + P(B)$$

Definition 2: A σ -algebra is a set $\mathcal{F}$ of subsets of Ω with the following axioms:

$$(i) \phi, \Omega \in \mathcal{F}$$

$$(ii) \text{ If } A \in \mathcal{F}, \text{ then } A^c \in \mathcal{F}$$

$$(iii) \text{ If } A_1, A_2, \dots, \in \mathcal{F}, \text{ then } \bigcup_{k=1}^{\infty} A_k, \bigcap_{k=1}^{\infty} A_k \in \mathcal{F}$$

Clearly $A^c := \Omega - A$ is the complement of A .

Definition 3. Stochastic process: A stochastic process $X(t)$ is a relations of random variables $\{X_t(\gamma), t \in T, \gamma \in \Omega\}$, i.e, for each t in the index set T , $X(t)$ is a random variable. Now we understand t as time and call $X(t)$ the state of the procedure at time t . In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

2.1.1 Problem Formulation

Here, we have two-asset correlated constant Geometric Brownian Motion model, the joint dynamics of Access Bank and First Bank market efficiency with constant and seasonal initial prices.

$$dS_t^{(1)} = \mu_1 S_t^{(1)} dt + \sigma_1 S_t^{(1)} dW_t^{(1)} \quad (1.1)$$

$$dS_t^{(2)} = \mu_2 S_t^{(2)} dt + \sigma_2 S_t^{(2)} dW_t^{(2)} \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) \quad (1.2)$$

$$S_0^{(1)} = 18 + 3 \cos(\omega t), S_0^{(2)} = 12 + 2 \cos(\omega t) \quad (1.3)$$

where $S_0^{(1)} = 18 + 3 \cos(\omega t), S_0^{(2)} = 12 + 2 \cos(\omega t)$ represents independent initial prices of Access and First, μ_1 represents drift coefficient of Access Bank, σ_1 represents volatility coefficient of Access Bank, μ_2 represents drift coefficient of First Bank, σ_2 represents volatility coefficient

of First Bank, ρ represents correlation, A, B are seasonal amplitude, ωt is seasonal frequency

$W_t^{(1)}, W_t^{(2)}$ represents independent Brownian Motions and t represents time.

2.1.2 Method of Solution

The investment equation consist of a system of variable coefficient problem whose solutions are not trivial. We adopt Ito's lemma, let $f(S_t^{(1)}, t) = \ln S_t^{(1)}$ and taking the partial derivatives yields:

$$\left\{ \frac{\partial f}{\partial S_t^{(1)}} = \frac{1}{S_t^{(1)}}, \frac{\partial^2 f}{\partial S_t^{(1)2}} = -\frac{1}{S_t^{(1)2}}, \frac{\partial f}{\partial t} = 0 \right\} \quad (1.4)$$

According to Ito's theorem gives:

$$df(S_t^{(1)}, t) = \sigma_1 S_t^{(1)} \frac{\partial f}{\partial S_t^{(1)}} dW_t^{(1)} + \left(\mu_1 S_t^{(1)} \frac{\partial f}{\partial S_t^{(1)}} + \frac{1}{2} \sigma_1^2 S_t^{(1)2} \frac{\partial^2 f}{\partial S_t^{(1)2}} \right) dt \quad (1.5)$$

Substituting (1.4) into (1.5) gives the following:

$$df(S_t^{(1)}, t) = \sigma_1 S_t^{(1)} \frac{1}{S_t^{(1)}} dW_t^{(1)} + \left(\mu_1 S_t^{(1)} \frac{1}{S_t^{(1)}} + \frac{1}{2} \sigma_1^2 S_t^{(1)2} \left(-\frac{1}{S_t^{(1)2}} \right) + 0 \right) dt \quad (1.6)$$

simplifying gives

$$= \sigma_1 S_t^{(1)} \frac{S_t^{(1)}}{S_t^{(1)}} dW_t^{(1)} + \left(\mu_1 \frac{S_t^{(1)}}{S_t^{(1)}} - \frac{1}{2} \sigma_1^2 \frac{S_t^{(1)2}}{S_t^{(1)2}} \right) dt \quad (1.7)$$

$$= \sigma_1 S_t^{(1)} dW_t^{(1)} + \left(\mu_1 - \frac{1}{2} \sigma_1^2 \right) dt$$

$$df(S_t^{(1)}, t) = \left(\mu_1 - \frac{1}{2} \sigma_1^2 \right) dt + \sigma_1 dW_t^{(1)} \quad (1.8)$$

Integrating (1.8) from 0 to t gives

$$\int_0^t d \ln S_t^{(1)} = \int_0^t df(S_t^{(1)}, u) = \int_0^t \left(\mu_1 - \frac{1}{2} \sigma_1^2 \right) du + \int_0^t \sigma_1 dW_t^{(1)} du \quad (1.9)$$

$$\ln S_t^{(1)} - \ln S_0^{(1)} = \left(\mu_1 u - \frac{1}{2} \sigma_1^2 u \right)_0^t + \left(\sigma_1 dW_t^{(1)} u \right)_0^t$$

$$\ln \left(\frac{S_t^{(1)}}{S_0^{(1)}} \right) = \left(\mu_1 - \frac{1}{2} \sigma_1^2 \right) t + \sigma_1 dW_t^{(1)}$$

Taking the ln of the both sides gives

$$S_t^{(1)} = S_0^{(1)} \exp \left(\left(\mu_1 - \frac{1}{2} \sigma_1^2 \right) t + \sigma_1 dW_t^{(1)} \right) \quad (1.10)$$

This gives the general solution of the SDE. To find the particular solution

$$S_0^{(1)}(t) = 18 + 3 \cos(\omega t) \quad ,$$

$$\Rightarrow S_0^{(1)} \exp \left(\left(\mu_1 - \frac{1}{2} \sigma_1^2 \right) t + \sigma_1 dW_t^{(1)} \right) = 18 + 3 \cos(\omega t)$$

$$S_0^{(1)} = 18 + 3 \cos(\omega t)$$

$$S_0^{(1)}(t) = 18 + 3 \cos(\omega t) \exp \left(\left(\mu_1 - \frac{1}{2} \sigma_1^2 \right) t + \sigma_1 dW_t^{(1)} \right) \quad (1.11)$$

From (1.2), let $f(S_t^{(2)}, t) = \ln S_t^{(2)}$ and taking the partial derivatives yields:

$$\left\{ \frac{\partial f}{\partial S_t^{(2)}} = \frac{1}{S_t^{(2)}}, \frac{\partial^2 f}{\partial S_t^{(2)2}} = -\frac{1}{S_t^{(2)2}}, \frac{\partial f}{\partial t} = 0 \right\} \quad (1.12)$$

According to Ito's theorem gives:

$$df(S_t^{(2)}, t) = \sigma_2 S_t^{(2)} \frac{\partial f}{\partial S_t^{(2)}} (\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)}) + \left(\mu_2 S_t^{(2)} \frac{\partial f}{\partial S_t^{(2)}} + \frac{1}{2} \sigma_2^2 S_t^{(2)2} \frac{\partial^2 f}{\partial S_t^{(2)2}} \right) dt \quad (1.13)$$

Substituting (1.12) into (1.13) gives the following:

$$df(S_t^{(2)}, t) = \sigma_2 S_t^{(2)} \frac{1}{S_t^{(2)}} (\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)}) + \left(\mu_2 S_t^{(2)} \frac{1}{S_t^{(2)}} + \frac{1}{2} \sigma_2^2 S_t^{(2)2} \left(-\frac{1}{S_t^{(2)2}} \right) + 0 \right) dt \quad (1.14)$$

simplifying (1.14) gives

$$= \sigma_2 S_t^{(2)} \frac{S_t^{(2)}}{S_t^{(2)}} (\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)}) + \left(\mu_2 \frac{S_t^{(2)}}{S_t^{(2)}} - \frac{1}{2} \sigma_2^2 \frac{S_t^{(2)2}}{S_t^{(2)2}} \right) dt \quad (1.15)$$

$$= \sigma_2 S_t^{(2)} (\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)}) + \left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) dt$$

$$df(S_t^{(2)}, t) = \left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) dt + \sigma_2 (\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)}) \quad (1.16)$$

Integrating (1.16) from 0 to t gives

$$\int_0^t d \ln S_t^{(2)} = \int_0^t df(S_t, u) = \int_0^t \left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) du + \int_0^t \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) du \quad (1.17)$$

$$\ln S_t^{(2)} - \ln S_0^{(2)} = \left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) \Big|_0^t$$

$$\ln \left(\frac{S_t^{(2)}}{S_0^{(2)}} \right) = \left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right)$$

Taking the ln of the both sides gives

$$S_t^{(2)} = S_0^{(2)} \exp \left(\left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) \right) \quad (1.18)$$

This gives the general solution of the SDE. To find the particular solution

$$S_0^{(2)}(t) = 12 + 2 \cos(\omega t)$$

$$\Rightarrow S_0^{(2)} \exp \left(\left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) \right) = 12 + 2 \cos(\omega t)$$

$$\begin{cases} S_t^{(1)} = 18 + 3 \cos(\omega t) \left(\exp \left(\left(\mu_1 - \frac{1}{2} \sigma_1^2 \right) t + \sigma_1 W_t \right) \right) \\ S_t^{(2)} = 12 + 2 \cos(\omega t) \left(\exp \left(\left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho W_t^{(1)} + \sqrt{1-\rho^2} W_t^{(2)} \right) \right) \right) \end{cases}$$

, $\mu_1 = 0.15, \sigma_1 = 0.30, W_t^{(1)} = 1, t = 1, \omega_t = 2\pi t$
 $\mu_2 = 0.12, \sigma_2 = 0.25, W_t^{(2)} = W_t^{(1)} = 1, t = 1, \rho = 0.5$

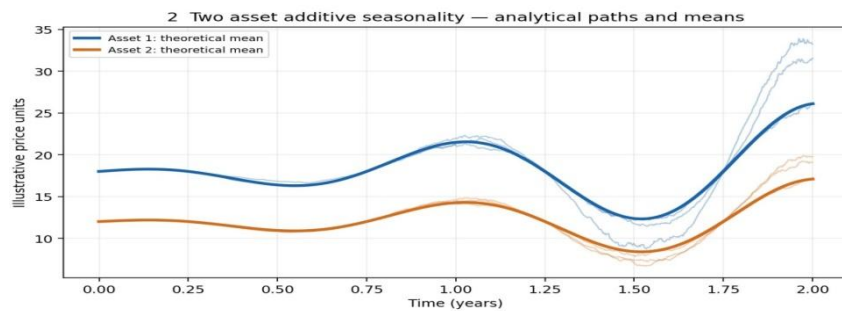


Figure 1: Three exact analytical paths per asset and theoretical expected means.

Figures 1 and 2 model a distressed bank under regulatory pressure where baseline valuation decays exponentially with multiplicative seasonality. Figure 1 shows three analytical paths and expected mean exhibiting up and down horizontal trend lines, indicating sideways trapped price with seasonal spikes but overall decay. Future Meaning: For a distressed bank, both volatility and drift lead to the same outcome - sideways decay. Changing drift (growth rate) does not help

$$S_0^{(2)} = 12 + 2 \cos(\omega t)$$

$$S_0^{(2)}(t) = 12 + 2 \cos(\omega t) \exp \left(\left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) \right) \quad (1.19)$$

Combining (1.11) and (1.19) gives complete solution of the systems

$$S_0^{(1)}(t) = 18 + 3 \cos(\omega t) \exp \left(\left(\mu_1 - \frac{1}{2} \sigma_1^2 \right) t + \sigma_1 dW_t^{(1)} \right) \quad (1.20)$$

$$S_0^{(2)}(t) = 12 + 2 \cos(\omega t) \exp \left(\left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) \right) \quad (1.21)$$

3.1 Results and Discussion

Two assets with additive seasonal components, with parameter values used for the simulations of graphical solutions.

because drift is negative due to regulatory pressure. Increasing volatility also does not give growth, it only increases the size of the up and down swings. Both sensitivity plots looking alike means the bank is insensitive to normal growth factors, it is under regulatory overhang. Future price will continue to oscillate horizontally and slowly decay until recapitalization is concluded.

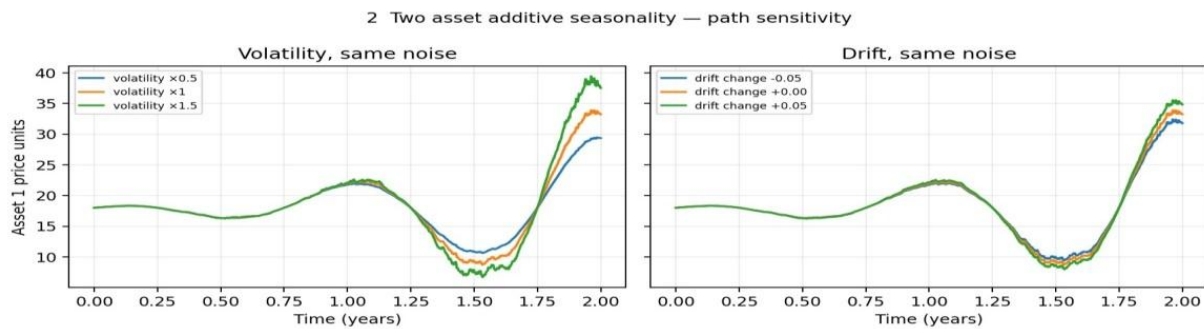


Figure 2: Asset Sensitivity using Identical Brownian increments

Figure 2: Three analytical paths per asset and theoretical expected means showing up and down horizontal trend lines due to multiplicative seasonality and decay path. This figure shows a distressed bank, not a growth bank. The baseline valuation is decaying, but the decay is not smooth. It shows up and down horizontal movements because of multiplicative seasonality. Future Meaning: The up and down horizontal trend means the share price is trapped in a sideways channel. It tries to rise due to seasonal news quarterly results, CBN forbearance, recapitalization rumors but falls back due to regulatory pressure and decay in fundamental value. The three analytical paths show high uncertainty, one path may go slightly up, one flat, one down, but all are oscillating around a falling expected mean. This is a classic distressed pattern, no clear trend, only noise.

4.1 Conclusion

This system extends Geometric Brownian Motion (GBM) to two correlated assets with constant initial prices captures comovement via correlation. Results show (i) with up and down horizontal lines means future price is trapped and decaying with seasonal hope rallies that fail. Decision is sell or avoid long-term, trade short-term only. (ii) where volatility and drift plots look alike and both show horizontal up and down means distressed bank is insensitive to normal growth drivers (iii) both volatility and drift lead to same sideways decay. Decision is do not buy for growth, use tight stop-loss for trading, and wait for regulatory clearance. (iv) Overall, Figures 1 and 2 represent a bank in regulatory distress - capital should be moved to stable banks like Access and First Bank until recapitalization succeeds.

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